The Ultimate Kitchen Renovation

I. UNIT OVERVIEW & PURPOSE:

This unit emphasizes the relationship among and real-world applications of quadrilaterals. It is essential that students explore practical applications of mathematics in the high school setting. This unit encompasses an overarching project that involves the student assuming the roles of both designer and contractor for a kitchen renovation. Throughout the process of the renovation, students will need to assess the area of the kitchen using scaled blueprints, design a tessellated tile pattern for the floor, and conduct a cost-analysis to determine a projected cost for the project.

Students will revisit the defining characteristics of quadrilaterals during the first lesson. Students then apply these concepts in addition to the concepts of scaling and proportions to determine the area and perimeter of a complex figure. The third lesson causes students to explore the relationship among quadrilaterals to create a tessellation. Using the information acquired through the first three lessons, students will conduct a cost analysis on the amount of money required to create their tessellation with tile flooring in a kitchen.

II. UNIT AUTHOR:

Sarah Dulaney, West Point Middle School, West Point
Melissa Harnist, Roanoke Valley Governor's School, Roanoke City
Shannon Royall Harrell, Patrick County High School, Patrick

III. COURSE:

Mathematical Modeling: Capstone Course (the course title might change)

IV. CONTENT STRAND:

Geometry

V. OBJECTIVES:

- The student will verify characteristics of quadrilaterals including parallelograms, rectangles, squares, rhombi, and trapezoids
- The student will use properties of quadrilaterals to solve real-world problems.

VI. MATHEMATICS PERFORMANCE EXPECTATION(s):

- MPE. 3: The student will use pictorial representations, including computer software, constructions, and coordinate methods, to solve problems involving symmetry and transformation. This will include
    - part d) determining whether a figure has been translated, reflected, rotated,

or dilated, using coordinate methods.

- MPE.4: The student will verify characteristics of quadrilaterals and use properties of quadrilaterals to solve real-world problems.

VII. CONTENT:

Students will renovate a kitchen using a budget and various materials that are on a client's wish list. Students will create a new kitchen floor design by tessellating various quadrilaterals. Students will have to find the area of the floor and counters in order to know how much of each material is needed. Students will have to find the area of each quadrilateral they use in their tessellation in order to ensure that they choose a tessellation pattern that covers the entire area.

VIII. REFERENCE/RESOURCE MATERIALS:

Throughout the course of this unit students will be using the following items:

- Blueprints for a kitchen - which are attached with all other supplementary items
- Computer access - Students will research the costs of materials to do this kitchen renovation (i.e. butcher's block, granite, laminate, tile, etc.) and use GeoGebra to create a tessellated floor design.

Other classroom activities and journal prompts are discussed in the context of the lesson or are attached in the supplementary items to this unit.
Van Hiele levels are often referred to in this unit. For more explanation regarding these levels, refer to the "van Hiele Levels Overview" document.

IX. PRIMARY ASSESSMENT STRATEGIES:

Reflections and journals, think/pair/share, class discussion, projects, and presentations.

X. EVALUATION CRITERIA:

Scoring rubrics are attached with the supplemental materials to this unit.

XI. INSTRUCTIONAL TIME:

Approximately 10-90 minute blocks or 20 - 45 minute class periods.

XII. Unit Outline:

Lesson 1 - Quadrilaterals

- Classroom activities:
  1. Quadrilateral challenge

2. Students will complete the quadrilateral flow and property charts that are enclosed.

- Journal entry
- Project

  Students will conduct a quadrilateral scavenger hunt throughout their neighborhood and take pictures of different quadrilaterals that they encounter in everyday life.

Lesson 2 - Area of Quadrilaterals

- Classroom activities:

  1. Students will derive area formulas using activities from NCTM Illuminations project
- Journal entry

Lesson 3 - Scaling and Proportions

- Classroom activities:
  1. Scaling and proportions activity
  2. Students will determine the square footage of the kitchen from a scaled blueprint drawing
- Journal entry

Lesson 4 - Artistic Tessellation

- Classroom activities:
  1. Students will create a tessellation that incorporates a specific set of polygons.
- Journal entry

Lesson 5 - Budgeting and Cost Analysis

- Classroom activities:
  1. Students will conduct a cost analysis to project the amount of money required to create their tessellation on the kitchen floor.
- Journal entry

Lesson 6 - Kitchen Tessellation and Presentation

- Classroom activities:
  1. Students will create a tessellation for the kitchen floor.
  2. Students will present the final product for this whole unit.

Lesson 1: Characteristics of Quadrilaterals

Strand

- Geometry

Mathematical Objective(s)

The goal of this lesson is to review fundamental concepts pertaining to quadrilaterals. These topics are typically taught in a high school geometry course, but serve as the foundation for this unit. These topics will be explored in greater depths with a particular emphasis on their applications in a real-world setting. More specifically, the student will be able to:

1. identify and distinguish among all types of quadrilaterals.
2. determine the area and perimeter of all types of quadrilaterals.

Mathematics Performance Expectation(s)

- MPE.4: The student will verify characteristics of quadrilaterals and use properties of quadrilaterals to solve real-world problems.

Related SOL

G.9 (Quadrilaterals and their applications)

NCTM Standards

- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships
    - analyze properties and determine attributes of two- and three-dimensional objects;
    - explore relationships (including congruence and similarity) among classes of two- and three-dimensional geometric objects, make and test conjectures about them, and solve problems involving them;

Materials/Resources

Describe the materials and resources (including instructional technology) you plan to use in each lesson.

- Protractor
- Straightedge/ruler
- Graph paper
- Flow chart

Assumption of Prior Knowledge

It is assumed that students have completed a geometry course prior to this lesson. Students will need to be familiar with and comfortable using the following terms: quadrilateral, parallelogram, trapezoid, rectangle, square, and rhombus, polygon, congruent, right angle, and bisect.

- Students will need to be comfortable using the following tools: ruler, protractor, and a compass.
- Students should be familiar with the structure and properties of polygons and quadrilaterals.
- Students should be operating at least at a level 2 on van Hiele scale with respect to the properties of quadrilaterals. Students who are not yet at this level may work with partners.

Introduction: Setting Up the Mathematical Task

In this lesson, students will investigate the relationship between categories of quadrilaterals and their properties. Students will also calculate the area of each type of quadrilateral.

Time outline: 1.5 90-minute block or 3 45-minute periods

- Quadrilateral Challenge: Approximately15 minutes
- Share results, sort and classify: Approximately 10 minutes
- Flow and properties chart: Approximately 20 minutes
- Lesson summary and reflection: Approximately 10
- Scavenger hunt challenge explanation: Approximately 10 minutes
- *Scavenger hunt presentation: Approximately 45 minutes
  *to be done at a later class period

The quadrilateral challenge and sharing activities is done in a think, pair, share format. Students should be placed with a partner or group of mixed abilities. The teacher may need to prompt students to be creative if there is a noticeable lack of a certain type of quadrilateral. The teacher can hint at different angles or different lengths of sides for students who are struggling to be creative.

Student Exploration 1: The Quadrilateral Challenge

Student/Teacher Actions:

- Students will begin with a challenge to create as many different types of quadrilaterals as possible in 15 minutes. They will use rulers, protractors, and graph paper to complete this task. By creating their own quadrilaterals, students will be forced to consider the different aspects of each shape: the angles, the lengths of each side, whether or not opposite sides are parallel, the lengths of the diagonals, etc.
- After creating the quadrilaterals, students are to sort their figures into specific groups. By doing so, the students will compare and contrast the attributes of each

shape. They will be able to build a hierarchy of properties. Students should justify their reasoning and record their rationale in their math journal.
- Students will share their quadrilateral drawing results with the class. We will take 10 minutes to sort and classify their results, looking for similarities and differences among their creations.
- After the sort, the teacher will facilitate a class discussion about the reasoning students used for grouping their quadrilaterals. This discussion should lead to the completion of the quadrilateral flow chart. The chart should include sketches of each type of quadrilateral as well as the specific properties that define each. The discussion and completion of the flow chart and properties chart may take up to 20 minutes.

Monitoring Student Responses

Student responses will be monitored through four distinctive methods for this exploration:

1. Students' discussions with their partner or groups as they create and sort their figures.
2. Students' journal entries where they justify their initial reasoning for their quadrilateral groupings and their responses to the other journal questions.
3. Classroom discussion that is facilitated by the teacher during the flow/properties chart.
4. Scavenger hunt findings.

Assessment

Journal/writing prompts

- What criteria did you use for your initial quadrilateral sort? Was it the most accurate way to sort the figures that you created? What modifications, if any, did you need to make in your groupings?
- What did you learn in today's lesson?
- What did you like about today's lesson?

Other: Scavenger hunt project.

The student will conduct a "scavenger hunt" for each type of quadrilateral discussed in today's lesson. Each quadrilateral is to be a unique representation of itself, meaning that a picture of a square object cannot count for a square, rhombus, and rectangle. Instead, each picture is to depict the defining properties of that particular quadrilateral that distinguishes it from other types. The student is to take a picture of the item and answer the following questions. No pictures from the internet are allowed. These pictures are to come from items encountered in the student's everyday life. Many of these items can be found around the house, whereas others may require the student to

go into a park or other public place. Students are to do a short presentation of their pictures where they will answer the following questions for each figure.

- Question 1: What is the item and where is it located?
- Question 2: What type of quadrilateral is shown in the picture?
- Question 3: List the defining features shown in the picture that make it classified as that specific quadrilateral.

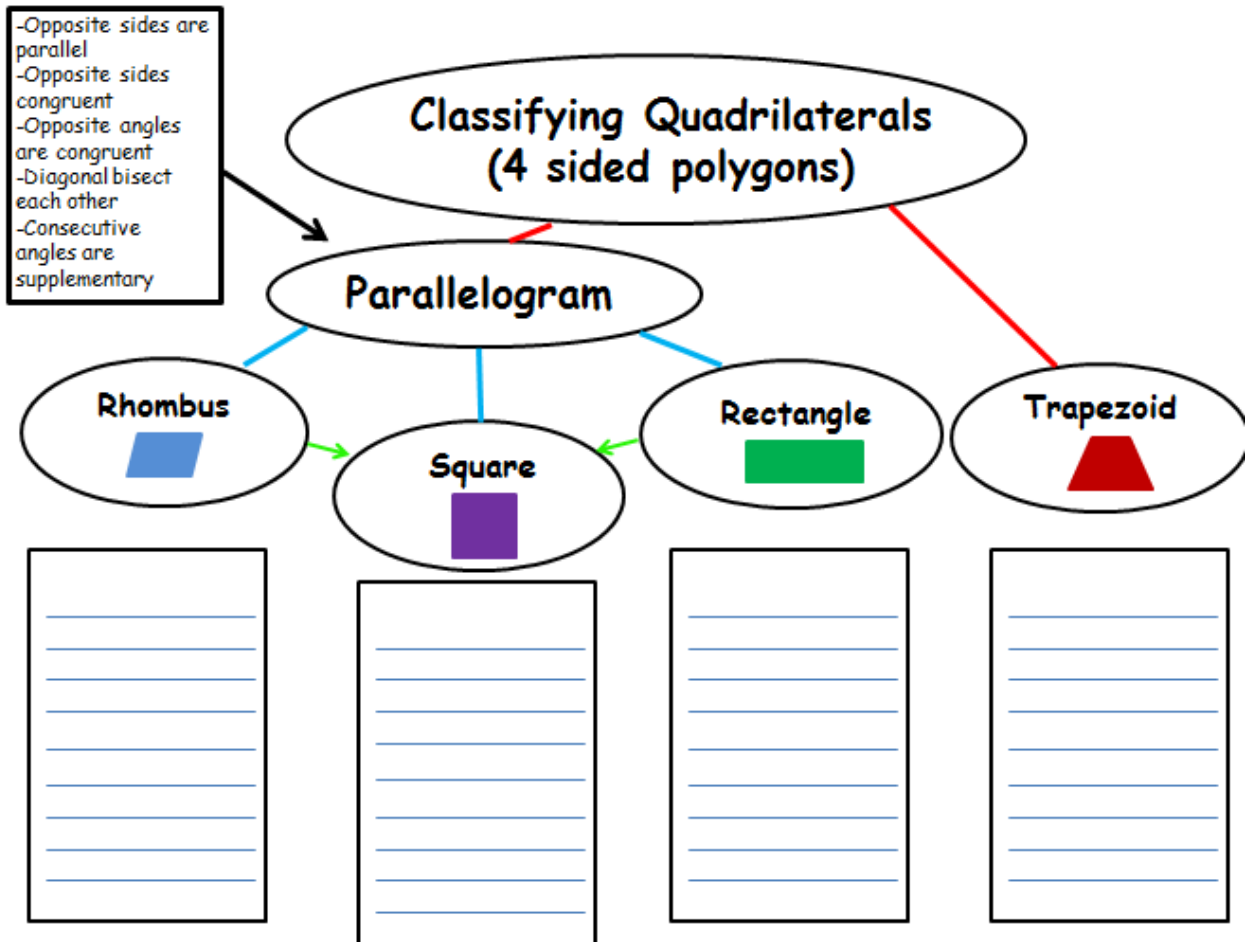
Extensions and Connections (for all students)

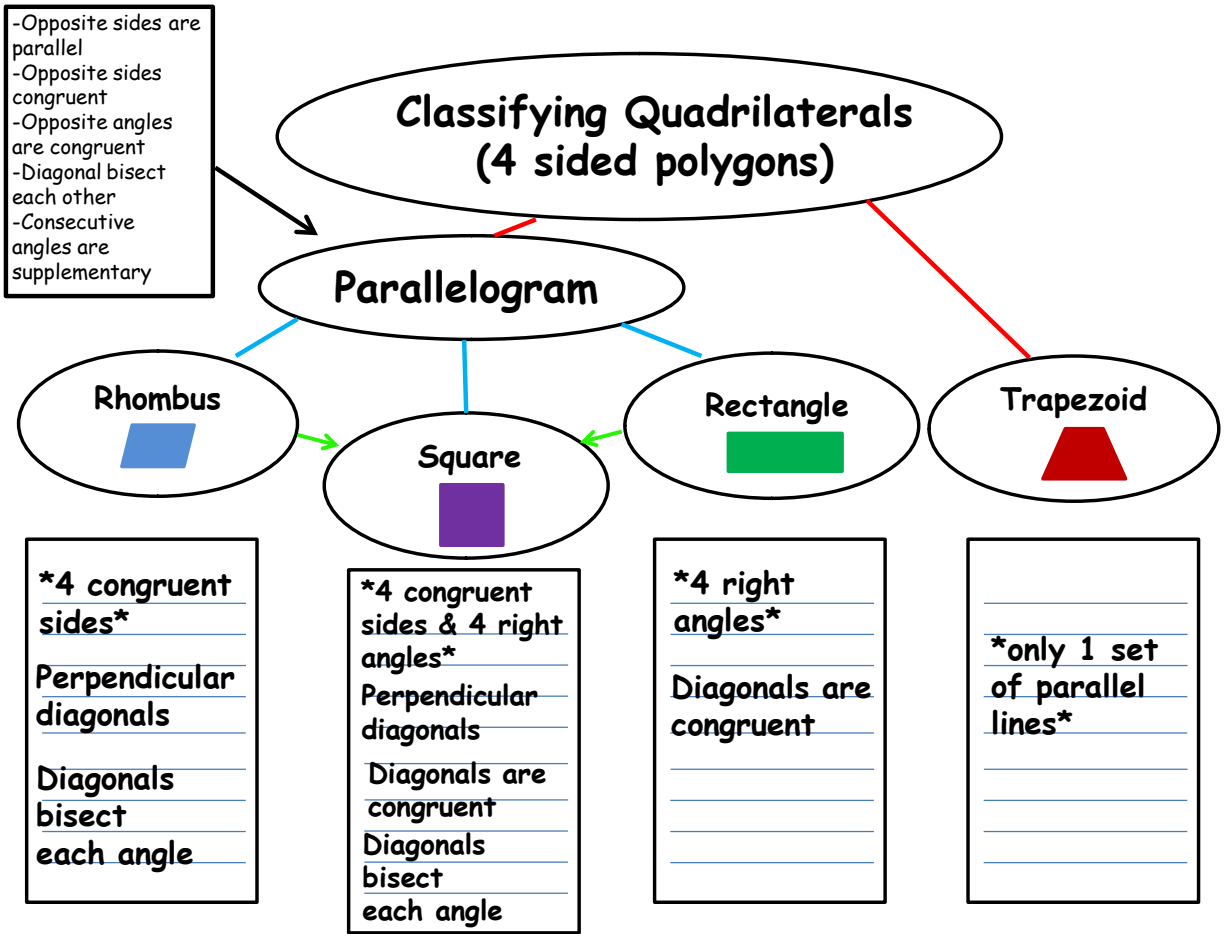
- Students will need approximately one week to find the quadrilaterals for their scavenger hunt. Students should present their findings at a later class period. This presentation could easily require one full class period depending upon the size of the class.
- This lesson has potential connections to architecture as students search for different quadrilaterals for the scavenger hunt.
- This lesson connects to Geometry.

Strategies for Differentiation

- The assessment piece of this lesson is differentiated by product. Students may present their scavenger hunt findings through a variety of means such as a collage, a flip chart, or a PowerPoint presentation. This will appeal to a variety of learning styles and also allow students of multiple ability levels to explore these concepts in varying depths by how creative they are in their search for the quadrilaterals.

Name: _____ Date: _____


-Opposite sides are parallel
-Opposite sides congruent
-Opposite angles are congruent
-Diagonal bisect each other
-Consecutive angles are supplementary
Classifying Quadrilaterals (4 sided polygons)
Parallelogram
Rhombus
Square
Rectangle
Trapezoid


-Opposite sides are parallel
-Opposite sides congruent
-Opposite angles are congruent
-Diagonal bisect each other
-Consecutive angles are supplementary
Classifying Quadrilaterals (4 sided polygons)
Parallelogram
Rhombus
Square
Rectangle
Trapezoid
4 congruent sides
Perpendicular diagonals
Diagonals bisect each angle
4 congruent sides & 4 right angles
Perpendicular diagonals
Diagonals are congruent
Diagonals bisect each angle
4 right angles
Diagonals are congruent
only 1 set of parallel lines

Name: _____ Date: _____

Quadrilateral Property	Parallelogram	Rectangle	Rhombus	Square	Trapezoid
All sides $\cong$					
Opposite sides $\cong$					
Opposite sides parallel					
Only one pair parallel sides					
All angles $\cong$					
Opposite angles $\cong$					
All angles are right angles					
Consecutive angles supplementary					
Diagonals $\cong$					
Diagonals perpendicular to each other					
Diagonals bisect each other					
One diagonal forms 2 $\cong$ Δ's Two diagonals form 4 $\cong$ Δ's					
Diagonals bisect 2 angles					

Quadrilateral Property	Parallelogram	Rectangle	Rhombus	Square	Trapezoid
All sides $\cong$			x	x	
Opposite sides $\cong$	x	x	x	x	
Opposite sides parallel	x	x	x	x	
Only one pair parallel sides					x
All angles $\cong$		x		x	
Opposite angles $\cong$	x	x	x	x	
All angles are right angles		x		x	
Consecutive angles supplementary	x	x	x	x	
Diagonals $\cong$	x	x	x	x	
Diagonals perpendicular to each other			x	x	
Diagonals bisect each other	x	x	x	x	
Diagonal forms 2 $\cong$ Δ's Two diagonals form 4 $\cong$ Δ's	x	x	x	x	
Diagonals bisect 2 angles			x	x	

Lesson 2 Title: Investigating Area Formulas

Strand

Geometry, Measurement

Mathematical Objective(s)

The goal of this lesson is to review the area formulas for quadrilaterals. The formulas are learned and applied in middle school math. However, the formulas are generally not derived from experience, but taken for granted. During this lesson, students will use manipulatives to support their reasoning while they derive the area formulas for each quadrilateral. The students will eventually use the area formulas to calculate the area of the spaces they are planning to remodel.

Mathematics Performance Expectation(s)

- MPE.4: The student will verify characteristics of quadrilaterals and use properties of quadrilaterals to solve real-world problems.
- MPE.7: The student will use similar geometric objects in two- or three-dimensions to
    - a) compare ratios between side lengths, perimeters, areas, and volumes

Related SOL

- G.9 (Quadrilaterals and their applications)

NCTM Standards

- Understand measurable attributes of objects and the units, systems, and processes of measurement
    - make decisions about units and scales that are appropriate for problem situations involving measurement
- Apply appropriate techniques, tools, and formulas to determine measurements.
    - analyze precision, accuracy, and approximate error in measurement situations;
    - understand and use formulas for the area, surface area, and volume of geometric figures, including cones, spheres, and cylinders
    - use unit analysis to check measurement computations.

Materials/Resources

- Ruler
- Protractor
- Graph paper
- Scissors
- Calculator

- Documents from NCTM Illuminations (http://illuminations.nctm.org/LessonDetail.aspx?ID=U160)
- Construction paper
- Projector
- Tangram Templates

Assumption of Prior Knowledge

It is assumed that students have encountered the area formulas for the following figures: triangles, rectangles, squares, parallelograms, and trapezoids. It is also expected that students have calculated areas of complex figures by subdividing the figure into shapes that have area formulas. Students should be familiar with the concept of area being the number of square units occupied by a two-dimensional figure. Students should be comfortable using protractors and rulers, measuring in either metric or standard US units.

Students should be operating at the Analysis level on van Hiele scale with respect to the properties of quadrilaterals. This will ensure proper derivation and application of the area formulas.

Students may have trouble calculating areas for complex shapes. These will be more involved than the simpler quadrilaterals that have consistent formulas. Students will need to be careful in calculating missing measurements before subdividing the figures into triangles or special quadrilaterals.

Students should have reviewed the hierarchy of quadrilaterals and should understand the relationships between the properties of each. This lesson builds on the analysis and abstraction levels. Students will make more complex connections between the properties of each type of quadrilateral as they make measurements and calculate areas. Students will become more comfortable with the relationships between the different quadrilaterals and use this information to more efficiently calculate measurements. The calculation of area is important in all areas of design and engineering.

Introduction: Setting Up the Mathematical Task

- In this lesson, students will investigate the development of the area formulas for all quadrilaterals. Students will then calculate the areas of different types of quadrilaterals. They will first calculate specific quadrilaterals, and then they will calculate area of composite shapes.
- Time outline: 2 90-minute blocks or 4 45-minute periods

    - Warm-up: 10 minutes
    - Describe components of area: 5 minutes
    - Verify square and rectangle area: 10 minutes
    - Parallelogram formula: 10 minutes
    - Trapezoid formula: 10 minutes
    - Complex figure discussion: 25 minutes
    - Tangram Activity with areas: 45 minutes

- Introduce the task.
    - Warm-up: From memory, write the formula for calculating area of each of the following: square, rectangle, parallelogram, and trapezoid. What do the variables stand for? How do you know that the formula works? Is there a general formula for all quadrilaterals? Why or why not? (Think-Pair-Share)

- Students will refresh their memories on area formulas for each shape. They will confirm the relationships between the heights and base lengths for each type of quadrilateral.

Student Exploration 1:

- Rectangles & Squares:
    - By using NCTM's "Squares and Rectangles Activity Sheet," students will calculate the area of four different rectangles. Students will draw conclusions about the formation of an array which is formed by multiplying the length and the width of the rectangle.
    - Students will then use their "Square and Rectangles Activity Sheet" to investigate the area of triangles. As suggested by NCTM, the students will draw diagonals on rectangles A, B, C, which will cut the rectangles in half - each side being a right triangle. The final investigation will be on rectangle D, which will be divided into two acute triangles by making two cuts instead of three.
    - Students will be split into two groups to experiment with making their own rectangles and triangles. Each group will use construction paper, protractors, and rulers to make a mixture of triangles (assigning each student a type, if necessary: right, obtuse, acute, scalene, isosceles, and equilateral) and rectangles (with varying ratios of length to width, including squares). The students will calculate the areas of the rectangles and triangles by applying the formulas.
- Parallelograms:

    - By using NCTM's "Rectangles and Parallelograms Activity Sheet," students will calculate the areas of the given rectangles. Then, by cutting off a corner - creating a right triangle through one of the vertices - this piece can be translated to the other side of the figure to create a parallelogram. Students will discuss why the area did not change.
    - Students will then do the reverse: cut right triangles off the last two pieces so that the translation to being a rectangle is shown.
- Trapezoids:
    - Then, students will be given lined paper, which is to be folded in half horizontally. They are to draw two parallel line segments of different lengths on the paper (it will be easier if there's more than an inch or two of space between the lines).
    - Students will then connect the endpoints of the line segments to form a trapezoid. The trapezoid will then be cut out, cutting through both layers of the paper. This creates two congruent trapezoids. Label the two bases and the height on each.
    - Rotate one of the trapezoids 180 degrees and lay the two trapezoids adjacent to each other to create a parallelogram.
    - Discuss how this visual matches the formula.
- Discussion: How are the formulas similar? How are they different? Why do these differences occur?
- Students will be working in pairs to work through each activity. They will discuss their responses and record their ideas together. After all students finish the activities, the teacher will lead a class discussion on their findings. During the activities, the teacher will be walking around, checking in with students, and answering questions.

Student Exploration 2: Tangrams!

<u>Part 1:</u>

- The teacher will facilitate a class discussion that consists of a summarization of student exploration 1: define formulas for each shape.
- Define complex figure as a figure that consists of two or more polygons. Complex figures are still polygons in that they are a closed-sided shape, but typically they cannot be classified as another type of polygon. For example, while a trapezoid can be decomposed into a combination of triangles and a rectangle, is not considered a complex figure because it can be classified as a specific type of quadrilateral.

<u>Part 2:</u> Practice decomposing figures

- Each student is to receive a template containing a complex figure and their own set of tangrams. The teacher is then to encourage students to decompose the figure using their tangrams. They are to trace their tangrams on to the template so that each polygon inside the complex figure can be seen easily. Students are to work independently on this activity with the teacher circulating the classroom. Once most students have completed their decomposition, the teacher is to facilitate a classroom discussion that emphasizes the different methods used to decompose the figure.

Part 3:

- Students will be placed in pairs. Each pair of students will be provided with a set of tangrams and a ruler. Students are to determine the area of each type of polygon present in their set of tangrams using the appropriate formulas and record these areas on a piece of paper.
- Students will then trade in their ruler for two templates of complex figures. Students are to use their tangrams to determine how to partition the figures into known polygons. They are to trace the tangrams that they selected onto the paper template containing the complex figure before removing them from the template. Students will then calculate the area of the complex figure using the areas of the known polygons. They will use this process to calculate the area of both complex figures.
- As students decompose the complex figure to determine its area, the teacher is to circulate the classroom and address problems as they arise. Students are to check with the teacher once they have found the area of the complex figure. Once both areas of the complex figures are found correctly, students will move into the assessment component of this exploration.

Part 4: Assessment (see below)

Assessment

· Describe and attach the assessments for each lesson objective.

- Exit ticket: Day 1, which concludes deriving the area formulas for rectangles, parallelograms, and trapezoids
    - How are the area formulas similar? How are they different?
    - Is it certain that these formulas will work for all shapes of a given category? Why or why not?
    - Construct quadrilaterals that meet a given area, such as 72 square units.

- Day 2: Using tangrams, construct a complex figure and trace its outline on grid paper. Trade papers with another student, whose job is then to reconstruct the figure with tangrams and find its area

Extensions and Connections (for all students)

· Lesson 4 of this unit builds on these concepts to extend them into the field of tessellations.

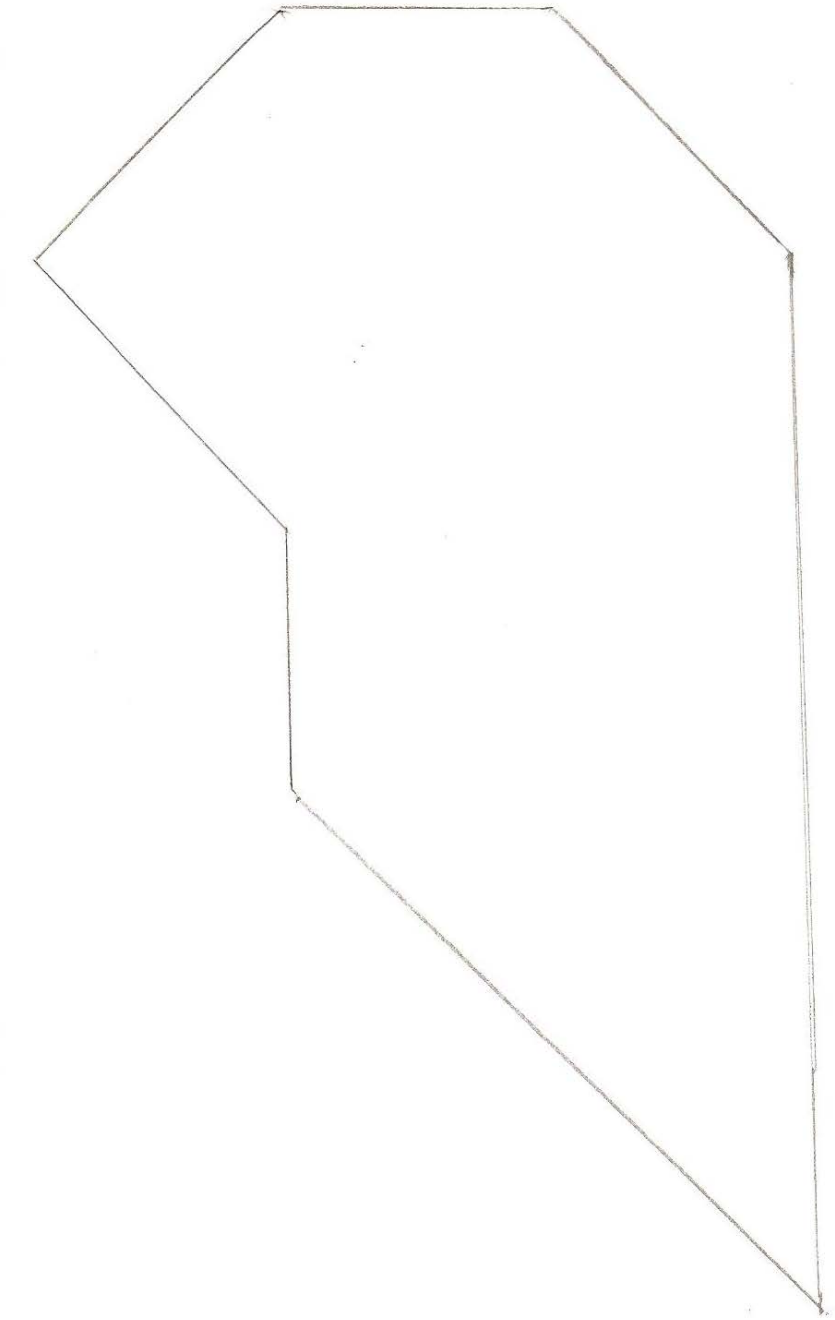
Strategies for Differentiation

A variety of learning styles are addressed through this lesson. Auditory learners are provided with ample opportunities to articulate their ideas through small group discussion. Visual and kinesthetic learners are able to use manipulatives. Students will see how the different area formulas interact with one another as they draw using the “Squares and Rectangles Activity Sheet” and as they work with tangrams later.

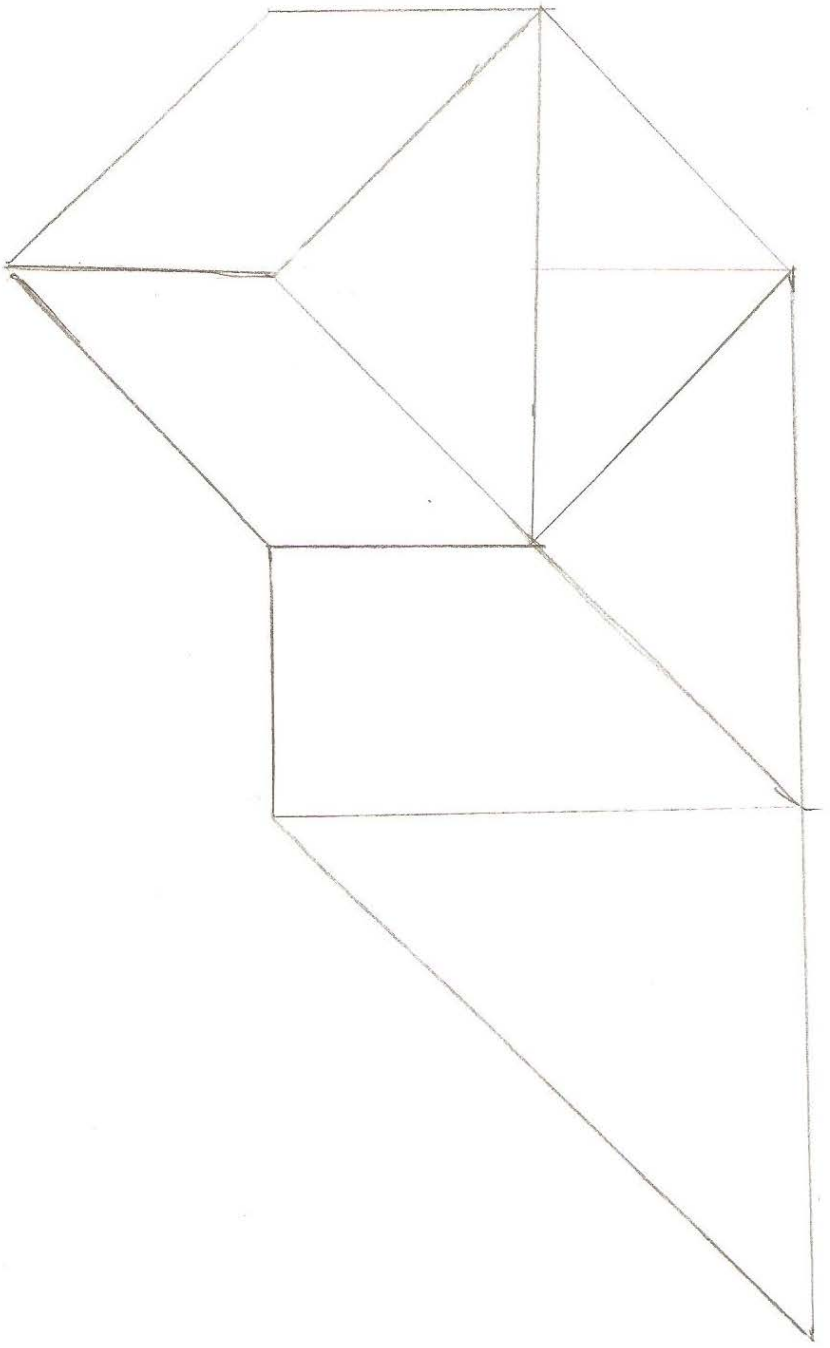
Teachers can easily differentiate this lesson to meet the needs of a mixed ability group of learners by developing other tangram templates for students to match. These templates could involve other types of transformations than those exhibited with the attached templates. Furthermore, should the enclosed templates be too sophisticated, these could be adapted easily to a simpler version to meet the needs of a student with less geometry skills.

Tangram Templates:

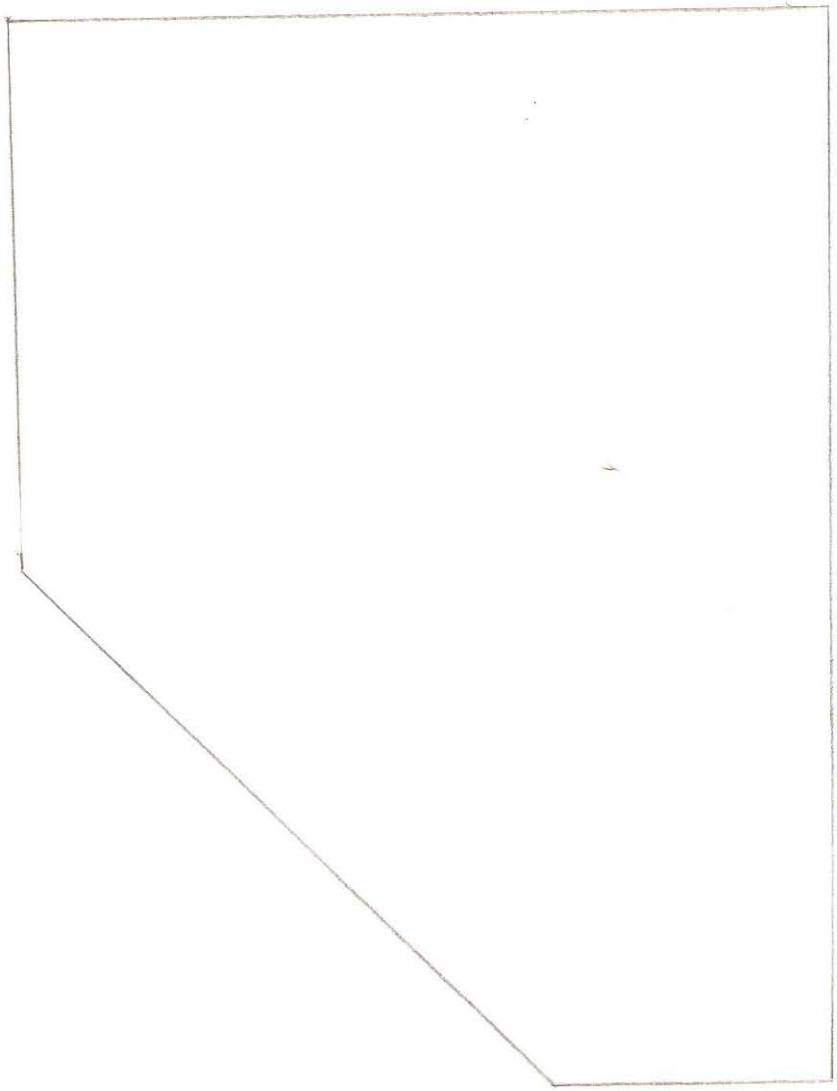
Template 1 Blank



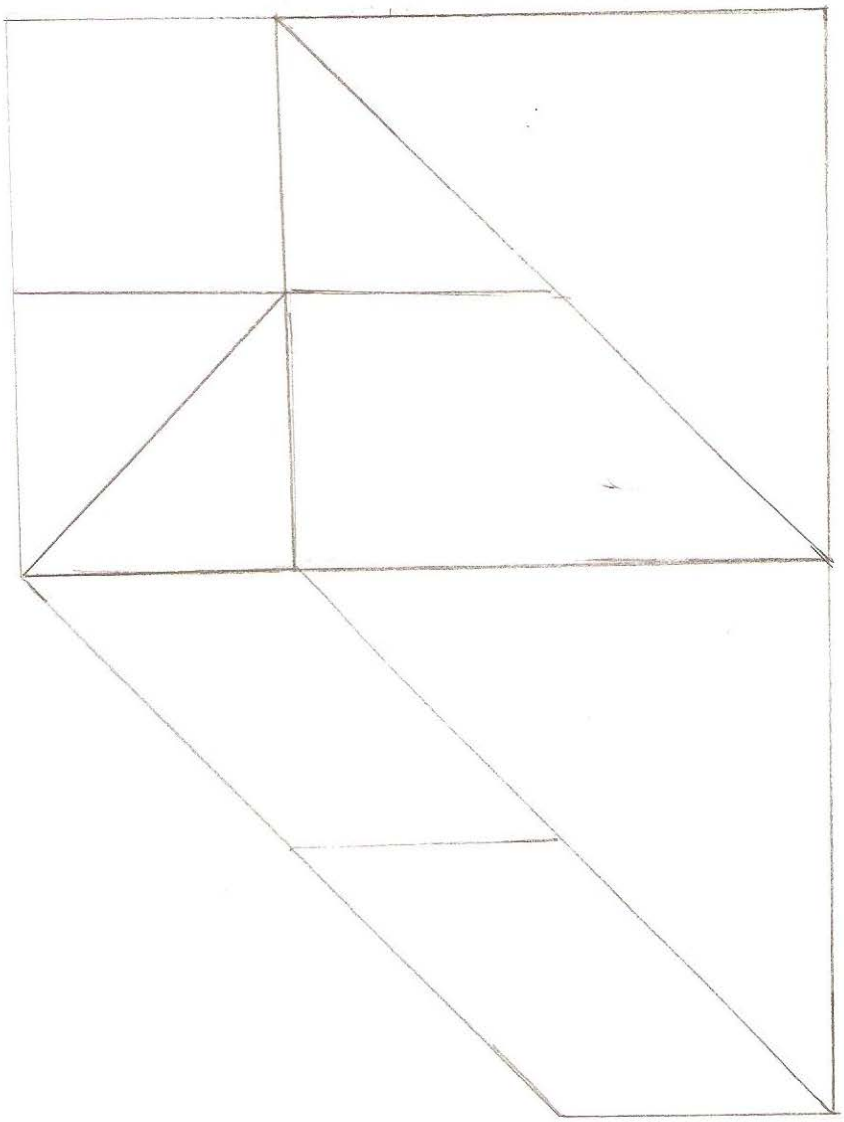
Template 1 Sample Solution (other solutions may exist)



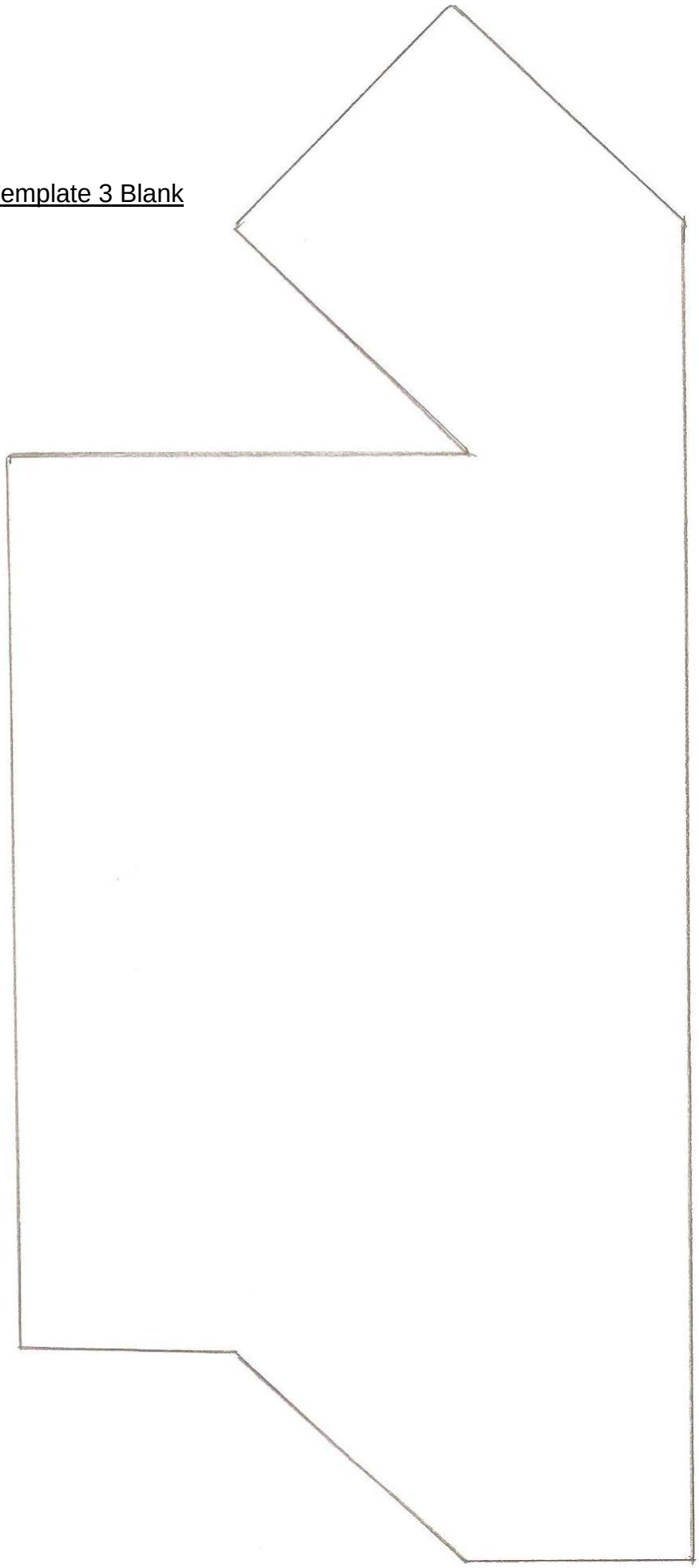
Template 2 Blank



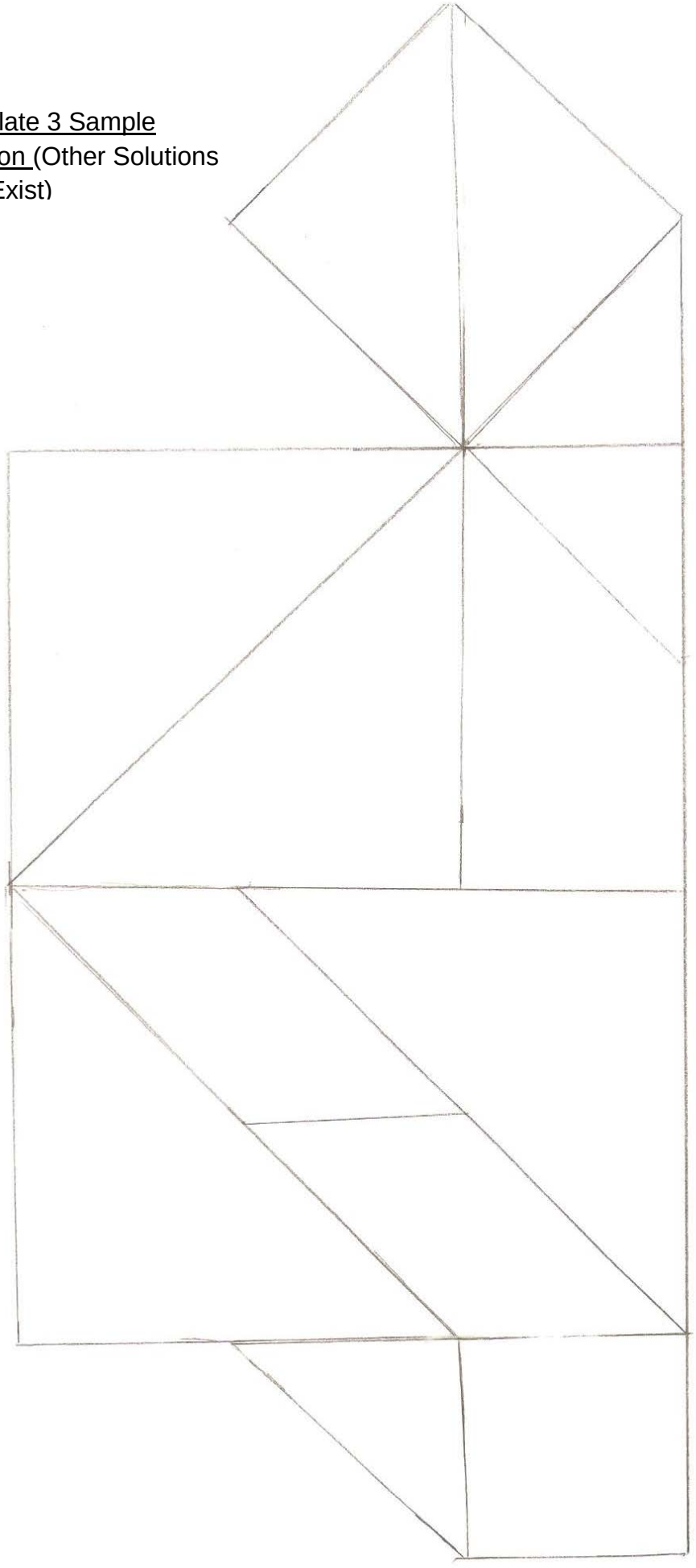
Template 2 Sample Solution (other solutions may exist)



Template 3 Blank



Template 3 Sample Solution (Other Solutions May Exist)



Lesson 3 Scaling and Proportions

Strand
Geometry

Mathematical Objective(s)
The goal of this lesson is to find the area of the kitchen floor and countertop for a kitchen renovation. Students will first review ratios and proportions. These topics are taught in elementary level mathematics courses and reviewed in high school geometry. After reviewing ratios and proportions students will find the area of a kitchen floor and counter space using a blueprint in order to renovate the given space.

Mathematics Performance Expectation(s)
MPE.1) The student will solve practical problems involving rational numbers (including numbers in scientific notation), percents, ratios, and proportions.
MPE.4) The student will verify characteristics of quadrilaterals and use properties of quadrilaterals to solve real-world problems.
MPE.7) The student will use similar geometric objects in two- or three-dimensions to
a) compare ratios between side lengths, perimeters, areas, and volumes;

Related SOL
G.9 (Quadrilaterals and their applications)
G.14 (3-Dimensional figures using ratio to compare area)
NCTM Standards

- Use visualization, spatial reasoning, and geometric modeling to solve problems
  - draw and construct representations of two- and three-dimensional geometric objects using a variety of tools
  - use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture
- Apply appropriate techniques, tools, and formulas to determine measurements
  - understand and use formulas for the area, surface area, and volume of geometric figures, including cones, spheres, and cylinders

Materials/Resources
Describe the materials and resources (including instructional technology) you plan to use in each lesson.

- Skittles, Starburst, and/or M & M's
- kitchen blueprint
- ruler
- calculator (if desired)

Assumption of Prior Knowledge

Students should already know how to find the area of a parallelogram, rectangle, rhombus, and square. Students are expected to divide the given space into shapes that they can recognize and find the area. Students should be familiar with solving and using proportions to find unknown variables.

Students should be operating on analysis level on the van Hiele scale with regard to quadrilaterals, ratios, and proportions. Since students will be operating on analysis level they will be able to use proportions to find unknown area.

Students may have difficulty decomposing the counter space and floor using the blueprint, in order to find the total area of each. Students must be comfortable finding the area of the shapes that he/she uses to decompose each. Also, students may struggle using the legend provided on the blueprint to find the actual area of the space. Students will have to understand how to set up a proportion using the legend and the measurements that they find when they complete their decomposition.

Similar figures should have already been discussed since the blueprint floor and counter space are similar to the actual area the students should renovate.

Introduction: Setting Up the Mathematical Task

In this lesson, students will discover the area of the floor and countertop by using the kitchen blueprint and given legend.

- Planned time outline: 1 90-minute block or 2 45-minute periods
    - Candy Ratio Warm-up : 10 minutes
    - Proportions Review: 15 minutes
    - Blueprint activity: 45 minutes
    - Review with a friend: 10 minutes
    - Journal entry: 10 minutes
- Introduce the task.
    - Review how to use ratios with Candy Ratio Warm-up. After this exercise, students should discuss how they think ratios and proportions can be used to solve real-world problems. After students have discussed their ideas with a partner, the class will discuss the different ways proportions can be used to solve real-world problems. After this class discussion, the students should work through some problems to review solving proportions.

Students will review ratios and proportions at the beginning of the lesson during the candy activity. After students have reviewed ratios and proportions, students will use proportions to find the area of the kitchen they are renovating. Using the provided blueprint, students will find the area of the floor and countertops. In order to accomplish this task, students will have to divide the given space up into quadrilaterals and then find the area of each. Once students have found the area of each quadrilateral, they will find the sum of the areas in order to find the area of the total space.

Student Exploration 1:

- Candy Ratio Warm-up: Students should pick a bag of Skittles, Starbursts, or M & M's (or any other candy that has multiple colors). After selecting their bag of candy students should count the total amount of pieces of candy.
- Have students answer the questions on the Candy Ratio WS (any of the following can be changed, edited, or deleted based on color availability for whatever candy is chosen). There is a **challenge question** for students moving through the lesson quickly, and the teacher should decide whether every child should answer it.

Monitoring Student Responses

Students should communicate their thoughts and knowledge throughout the lesson with one another and with the entire class during class discussions. After the candy ratio review, students are expected to share their thoughts with a classmate and write a brief summary of key ideas they remembered, learned, and/or discussed during and after the candy ratio review. Students should jot down their thoughts before moving on to finding the area using the blueprint, so that their ideas are fresh on their minds. This is also an opportunity for students to record any questions they may have that have not been answered or need further discussion.

Throughout this exploration the teacher will be walking around to answer any questions that may arise. Students that find this task difficult are free to discuss their problems with a partner (preferably) or the teacher.

Student Exploration 2:

Blueprint calculations:

- Pass out the blueprints and rulers to students and ask them to explore on their own before giving them any directions.
- After allowing students time to ponder, ask students the following questions:
    - How can you use ratios and proportions to find the area of the floor? The counter space?

    - How can you make this space easier to work with and find the area?
- Students should discuss their answers to these questions with a partner, then as a class.
- After the class discussion, students should use their blueprint and ruler (and calculator, if desired) to find the area of the counter and floor.

Monitoring Student Responses

There are multiple opportunities for students to communicate their thoughts with their classmates throughout this activity. Students are expected to discuss their plan with a partner, followed by a class discussion to ensure that all students have a good idea of how to approach this problem. Also, during the activity students are free to discuss with a classmate. During these discussions students will voice their thoughts and any problems they may be having, allowing other students the opportunity to answer their questions.

Throughout this exploration the teacher will be walking around to answer any questions that may arise. Students that find this task difficult are free to discuss their problems with a partner (preferably) or the teacher.

After students have had time to complete this activity, they should discuss their work with a classmate. Once each student pairing is confident with their calculated area, then they will turn in the area of the flooring and the countertop with the blueprint partitioned into the appropriate polygons to the teacher.

After all groups have turned in their area calculations and blueprints, the last few minutes will be a class discussion to ensure that all students were able to complete this activity to find the area of the counter space and floor by decomposing each area into quadrilaterals and triangles. This will guarantee that students will be able to complete the tessellation exploration in Lesson 4. Following this class discussion, students will write in their journals so that the teacher can make sure everyone will be ready for the next lesson.

Assessment

Tasks:

- Find the area of the floor and counter space using the kitchen blueprint and the given legend.
- Students will turn in their blueprint with their decomposition and area calculations which will be scored using the attached rubric.

Journal/writing prompts

- Please write at least one thing that you learned today and one question that you have after today's lesson.
- Please write your thoughts about today's lesson. What did you enjoy? What did you struggle with (if applicable)?

Strategies for Differentiation:

- Students that progress quickly through this lesson and have a higher level of skill may reevaluate the counter-top area to compensate for a sink that could be over the counter or under the counter. The choice can be left to the student as to how the sink will fall in relation to the counter.
- Also, students can find the area of the counter if the counters hang over the cabinets one inch since this will affect the area.

Name: _____ Date: _____

CANDY RATIO Worksheet

1. What type of candy did you choose? _____
2. How many blue pieces of candy do you have? How can you represent this as a ratio? _____
3. What is the ratio of yellow candies to orange candies? _____
4. If the ratio of red candy to green candy is 4:5, count how many red candies you have and determine how many green candies you would have using the given ratio.

   _____
5. Find the ratio of brown (or purple) candies to total candies. Now using the ratio you just found, determine how many brown (or purple) candies you would have in a bag with 200 total candies? 500 total? _____
6. **Challenge question** Suppose the ratio of green candies to red candies is 3:4. If there are 360 candies in the bag, how many of them are red? _____

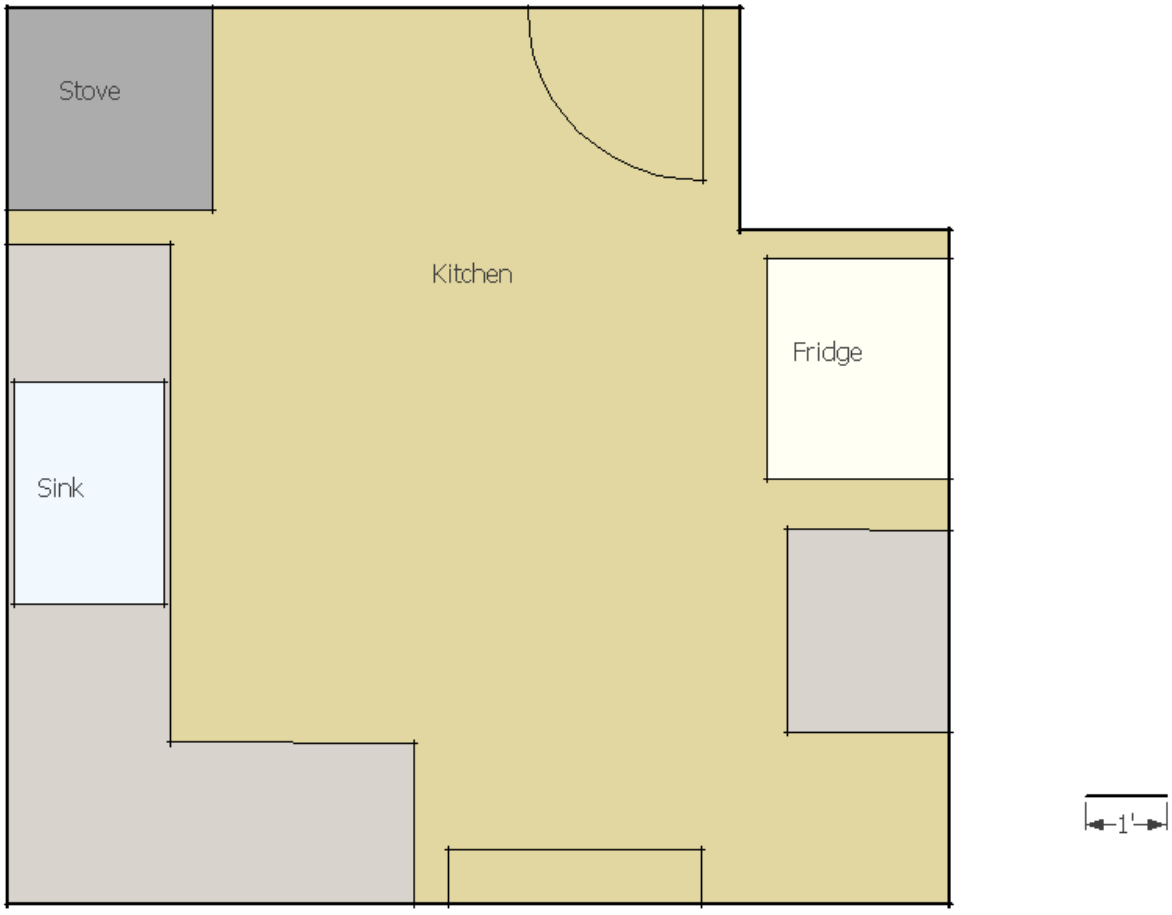
Name: _____ Date: _____

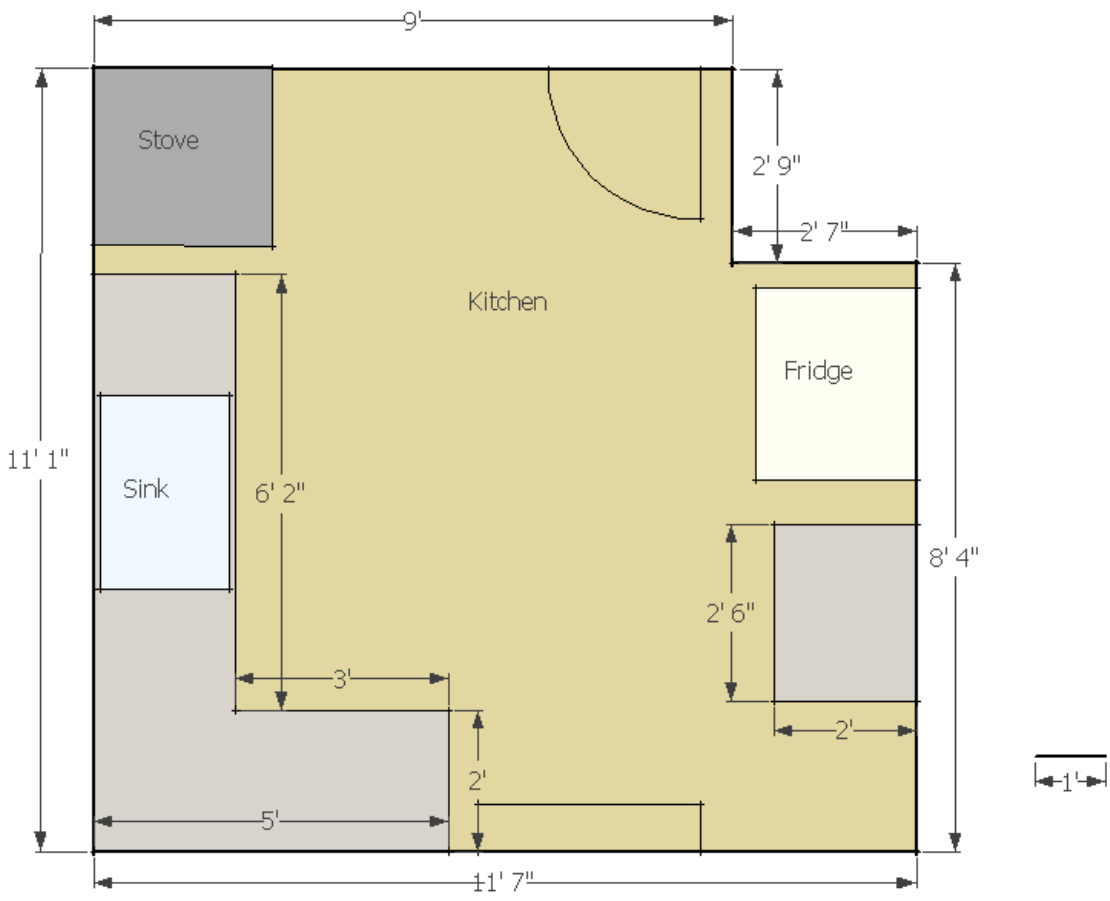
Rubric for Assessing Area Calculations for the Kitchen Floor and Counter space

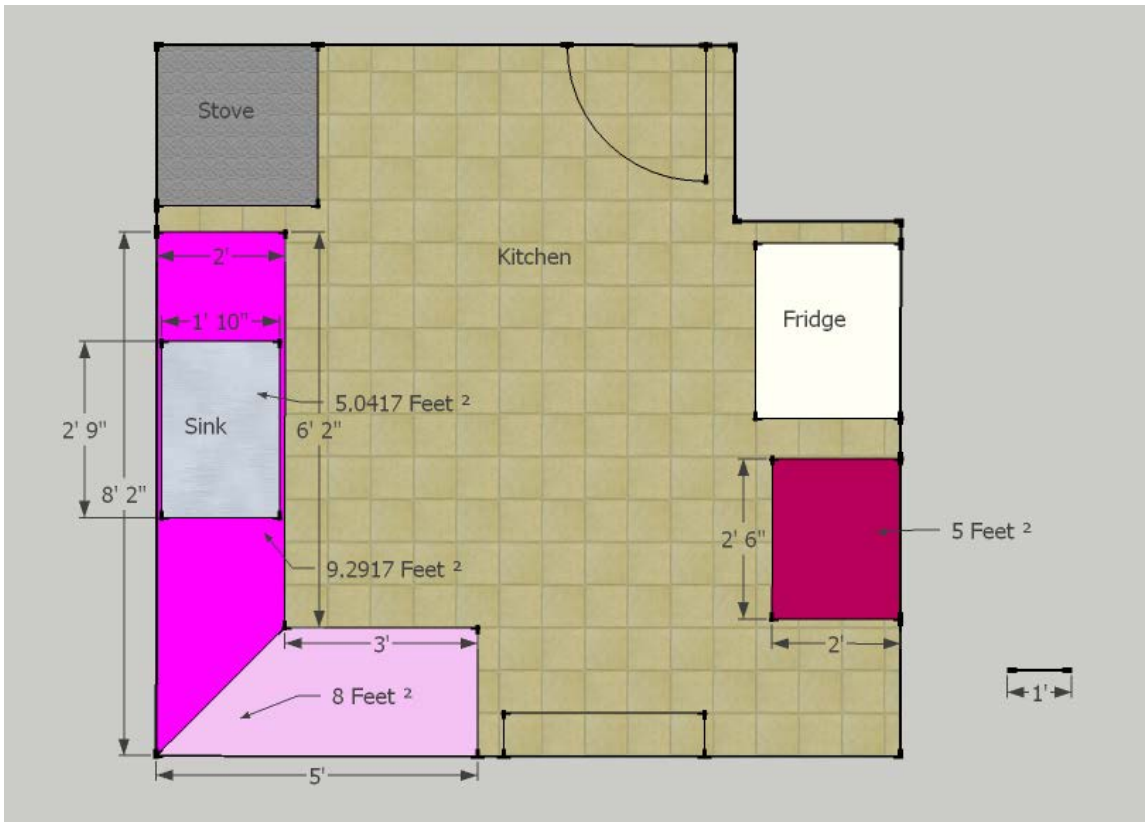
Criteria	Points Earned	Points Possible
Three of the following five polygons are used to subdivide the kitchen countertops and flooring: • triangle • square • rectangle • rhombus • trapezoid • parallelogram		10
The blueprint is clearly partitioned so that all of the polygons used to determine the area of the kitchen's floors and counter space are easily distinguishable.		5
The individual areas of the polygons are found using the appropriate formulas. This work should be attached.		30
The total area of the flooring and countertops is found using the sum of the individual polygons that make up the kitchen's components.		5
Appropriate “obstacles,” such as the undercount kitchen sink, are removed from the total area calculations		10
The scale factor is accounted for in calculations.		10
Units are included with values when appropriate.		5
Total		75

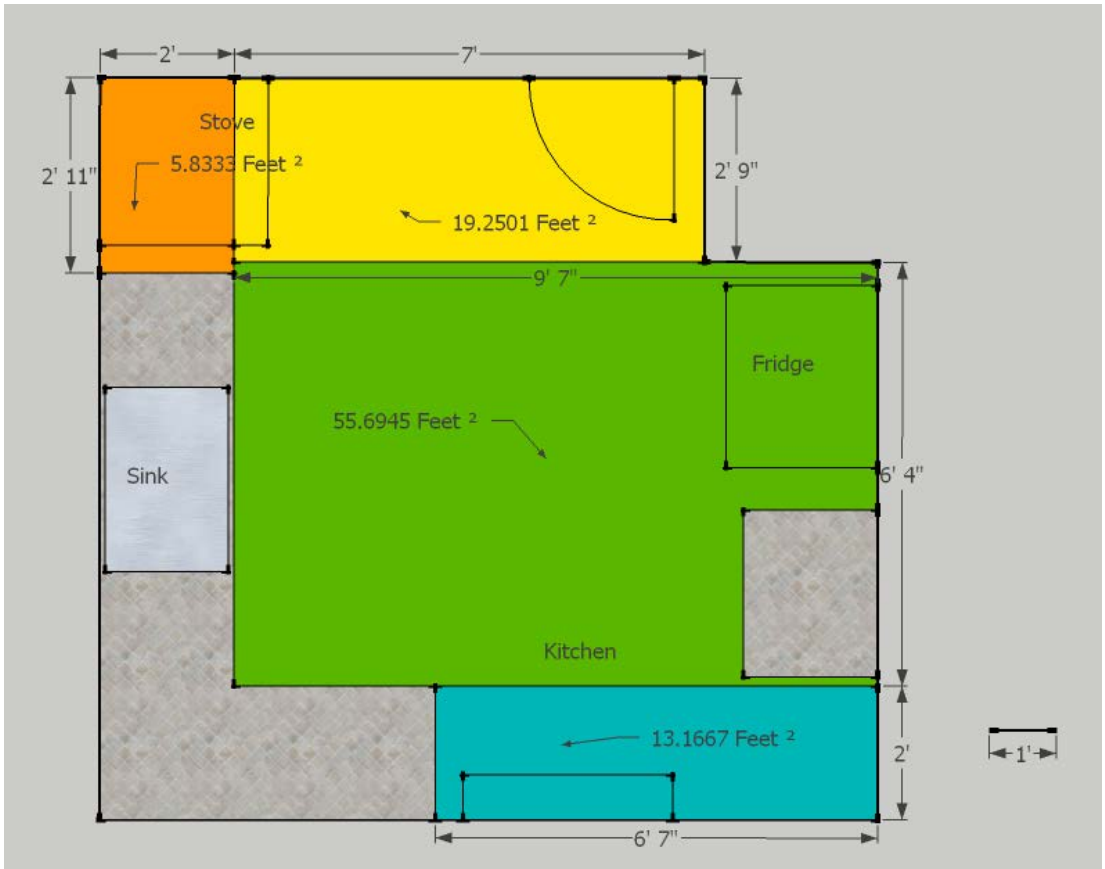
Name: _____ Date: _____

Kitchen Blueprints




9'
Stove
2' 9"
2' 7"
Kitchen
Fridge
11' 1"
Sink
6' 2"
8' 4"
2' 6"
3'
2'
2'
5'
1'
11' 7"


Stove
Kitchen
Fridge
2'
1' 10"
Sink
5.0417 Feet ²
6' 2"
2' 9"
8' 2"
9.2917 Feet ²
2' 6"
5 Feet ²
3'
2'
8 Feet ²
1'
5'


2'
7'
Stove
2' 11"
5.8333 Feet ²
19.2501 Feet ²
2' 9"
9' 7"
Fridge
55.6945 Feet ²
Sink
6' 4"
Kitchen
13.1667 Feet ²
2'
1'
6' 7"

Lesson 4: Artistic Tessellations

Strand
Geometry

Mathematical Objective(s)

The goal of this lesson is to review tessellations with students. Students will design a tessellation using at least two polygons to cover the entire space. The student is expected to make this exploration as creative as possible; therefore there is a minimum of using two colors when designing their tessellation.

Mathematics Performance Expectation(s)
List all applicable **Virginia College and Career Ready Mathematics Performance Expectation(s)**

- MPE.3: The student will use pictorial representations, including computer software, constructions, and coordinate methods, to solve problems involving symmetry and transformation.
- MPE.4: The student will verify characteristics of quadrilaterals and use properties of quadrilaterals to solve real-world problems.

Related SOL

- G.3 (Pictorial representations to solve problems involving symmetry and transformation)
- G.9 (Using properties of quadrilaterals to solve real-world problems)
- G.10 (Real-world problems involving angle measures, including tessellations)

NCTM Standards

- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships
    - explore relationships (including congruence and similarity) among classes of two- and three-dimensional geometric objects, make and test conjectures about them, and solve problems involving them
- Apply transformations and use symmetry to analyze mathematical situations
    - understand and represent translations, reflections, rotations, and dilations of objects in the plane by using sketches, coordinates, vectors, function notation, and matrices
- Use visualization, spatial reasoning, and geometric modeling to solve problems
    - draw and construct representations of two- and three-dimensional geometric objects using a variety of tools

    - use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture
- Understand measurable attributes of objects and the units, systems, and processes of measurement
    - make decisions about units and scales that are appropriate for problem situations involving measurement

Materials/Resources

Describe the materials and resources (including instructional technology) you plan to use in each lesson.

- Ruler
- Scissors
- Colored pencils, crayons, or markers
- 8 ½" by 11" graph paper
- 8 ½" by 11" computer paper (unlined)
- Tangram template (for students with special needs)

Assumption of Prior Knowledge

- Students should be familiar with the basics of tessellations - what they are and how they can be constructed. They should know how to make sure that the angle measurements of each shape in a tessellation will fit together such that there will be no gaps or overlap between the pieces. Students should be able to have a discussion on which regular polygons can tessellate and which ones cannot. From this discussion, they should also discuss how irregular polygons would have to be constructed in order to tessellate.
- Students should be comfortable with transforming polygons by reflection, rotation, and translation in order to create a tessellation. Students should already be familiar with finding the area of polygons. Dividing complex figures into polygons, so that the area of the region can be found.

Introduction: Setting Up the Mathematical Task

· Clearly introduce the goal of the lesson.

- In this lesson, students will create a tessellation using quadrilaterals and other polygons. It will be introduced in relation to M.C. Escher's famous tessellations with the goal being for the students to create their own artistic tessellations. Later, in lesson 6, students will create a tessellation using the illustrations of the floor plan of the kitchen.

Describe planned time outline: 1 90-minute block or 2 45-minute periods

- Tessellation Activity : 45 minutes
- Tessellation Paper: 45 minutes

- Introduce the task.
  - Show the “Create a Tessellation” video by teachertubemath on YouTube (http://www.youtube.com/watch?v=_Lg19NvW_sU)
  - Students will review tessellations before creating their tessellation renovation design. This tessellation activity will be a fun way for students to review tessellations before having to worry about area or complex figures, since their objective is to tessellate at least two polygons to cover a 8 ½” by 11” sheet of paper. After completing this activity, the student will analyze and communicate their thoughts about tessellations in their reflective paper.

Student Exploration 1: Art Tessellation

- Some of the most famous artists are also some of the most famous mathematicians. This is because art often incorporates a significant amount of mathematics in its design. M.C. Escher is known for his unique art pieces involving optical illusions, impossible figures, and tessellations. He is also regarded as a mathematician because of his use of geometric properties involving parallel lines, reflections about a line of symmetry, and translations of polygons patterns.

- The students’ task is to create their own piece of art: a tessellation! They are to use a minimum of 2 different kinds of regular polygons. Students are to create a template of their polygons on the graph paper and cut out these polygons to tessellate on the 8 ½” by 11” sheet of unlined paper. Since the coloring of the tessellation is part of what makes them fun to look at, students are to color their tessellation using a minimum of two different colors. Students are to complete this activity individually.

After completing their tessellation, each student is to write a one page reflection that answers the following questions. The write-up needs to be free of spelling and mechanical errors.

Reflection questions:

1. What is a tessellation?
2. In a tessellation, where do the polygons meet? Why is this important?
3. What makes your picture a tessellation? What shapes did you use?
4. How can one determine if one particular regular polygon will tessellate?

5 How can one determine if a combination of shapes will tessellate such as a square and a triangle, a square and a hexagon, etc.

Monitoring Student Responses

Students will demonstrate their knowledge about tessellations during this activity and after in their reflection paper. Each student's level of understanding will determine how many polygons will be used to tessellate the plane without gaps or overlapping. Also, students' answers to the above questions will demonstrate their knowledge about tessellations. Students will communicate their thinking from this activity in their one page reflection that should be turned in after it has been completed.

Throughout this exploration the teacher will be walking around to answer any questions that may arise. Students that find this task difficult are free to discuss their problem(s) with the teacher. The student should talk out their misconceptions) and work through their problem(s). In other words, the teacher will help the student come up with a solution, as opposed to just answering the question.

Extensions and Connections (for all students)

- This lesson has extensions in the arts in that students could explore M.C. Escher's tessellations and discover how he used polygons as the template for repeated artistic patterns.
- This lesson will be expanded upon further in lesson 6 when students apply their knowledge of tessellations to their kitchen renovation project.

Strategies for Differentiation

- This lesson is naturally differentiated for a diverse range of ability students in that gifted students could consider how to tessellate multiple polygons as opposed to just two. Furthermore, students that are gifted more artistically than mathematically are provided with the opportunity to demonstrate their skills. The fact that students are creating their own tessellations allows them to express their creativity and challenge themselves at a level that is best suited to their abilities.
- Students can create polygons using graph paper to have a template to use to create their tessellation.
- Students with disabilities will be provided tangrams to use to tessellate the plane.

Name: _____ Date: _____

Tessellation Project Rubric

Description	Points Earned	Points Possible
Tessellation: · 2+ different polygons used – 2 points · Fills 8 ½ by 11 sheet – 2 point · 2+ colors used – 2 points · Tessellates around a point – 4 points		10
Reflection: · Response to question #1– 2 points · Response to question #2 – 4 points · Response to question #3 – 4 points · Response to question #4 – 4 points · Response to question #5 – 4 points · Free of grammar and mechanical errors – 2 points		20
Totals:		30

Lesson 5: Budgeting and Cost Analysis

Strand
Number and Operations, Geometry

Mathematical Objective(s)
The goal of this lesson is to investigate the cost of implementing a tile design for the floor. Students will look at the cost of various materials to complete the renovation plans for the kitchen. They will need to figure out how many of a given tile will be needed to cover the calculated area. Students will eventually present their designs and calculations for cost given the materials chosen. Students will be expected to fit their plans within a given budget. The teacher should set a budget that is most appropriate for the socio-economic status of his/her community and is relevant to the cost of materials. To gain greater insight into the cost of materials, the teacher should do preliminary research at local home improvement stores in order to set a realistic budget.

Mathematics Performance Expectation(s)
- MPE.1: The student will solve practical problems involving rational numbers (including numbers in scientific notation), percents, ratios, and proportions.

Related SOL
- EPF. 10c (personal finance - decision making skills)

NCTM Standards
- Understand measurable attributes of objects and the units, systems, and processes of measurement
    - make decisions about units and scales that are appropriate for problem situations involving measurement
- Apply appropriate techniques, tools, and formulas to determine measurements
    - analyze precision, accuracy, and approximate error in measurement situations
    - use unit analysis to check measurement computations

Additional Objectives for Student Learning (include if relevant; may not be math-related):
- CTE 16: Demonstrate job-specific mathematics skills http://www.cteresource.org/verso/courses/8515/building-trades-i-tasklist
    - using mathematical reasoning and processes to accomplish job-specific tasks (e.g., using geometry and algebra to predict required supplies for a construction job, using computer mathematics to create a programming algorithm)

    - making calculations related to personal finance (e.g., wage rates, paycheck deductions, taxes).
- English 11.1 or 12.1
- English 11.8

Materials/Resources

- Samples of tiles, typically offered from home improvement stores
- Catalog of kitchen improvement supplies
- Computer with internet access
- Calculator
- Blueprint
- Tessellation design
- Record sheet for research

Assumption of Prior Knowledge

Students should be operating on Analysis level on van Hiele scale with respect to researching various materials, applying costs to amount needed, and budgeting. This means that they will need a fundamental understanding of number sense in relation to money. Students will need to be familiar with using the internet for research as well. Also, students should be comfortable with citing their sources in order to include their sources in their presentation in lesson 6.

Introduction: Setting Up the Mathematical Task

- In this lesson, students will research construction materials to get an idea on what is available for the designs they have yet to make. They will have to calculate how many of each type of tile they will need to cover the given areas for both the floor and the countertops. The students will be expected to fit their costs into a certain budget.
- Describe planned time outline: 1 90-minute block or 2 45-minute periods
    - Research: 45 minutes
    - Cost analysis with partner: 45 minutes

· Introduce the task.

- Students will discuss what supplies would be needed for any basic renovation, such as caulk, grout, measuring tape, etc. These will be assumed to be a covered cost and will not affect the students' budget. Students will also make predictions on whether it would be more cost efficient to buy larger tiles or smaller tiles to fit their designs. They will discuss experiences, if any, regarding any type of home renovation. The question should also be raised about what to

do if tiles won't fit the design perfectly - there can be a few associated with having to cut tiles to size, but it is a possibility that should be discussed.
- If possible, allow students to check out tile samples so that they can get a feel for size, material, texture, and price per square unit.

Student Exploration 1: Research

- Students may research tile cost either through catalogs or through a home improvement store's website. During the initial research, students will record at least three options for each type of tile used in their tessellations using the given record sheet.
    - Students will make a record of the material, color, and brand of the tile as well as its dimensions and price
    - Students will also calculate how many of each tile will be needed to cover the area, and will then calculate the total cost for purchasing the set of tiles
- Students should be given sufficient time to browse multiple sources.
- Students will then decide which combination of tiles is both pleasing and affordable.
- For the tiles chosen by the students:
    - If found online, pictures should be saved so that they can be later incorporated into the presentation
    - If found in a catalog, pictures should be copied or scanned so that they can later be used in the presentation
- Students should research materials on their own by gathering data about different materials for the floor and counter space.
- They can work on the cost analysis with a partner in order to find the cost based on how much of each material is needed.
- Once tiles have been chosen, students should use the scale factor to figure out how large the tiles will appear once included in the blueprint.

Student/Teacher Actions:

- Throughout this exploration the teacher will be walking around to ensure that all students are staying on task while researching in the computer lab. Also, the teacher can answer any questions that may arise. Students that find this task difficult are free to discuss their problem(s) with the teacher. The student should talk out their misconception(s) and work through their problem(s). In other words, the teacher will help the student come up with a solution, as opposed to just answering the question.

- Students will use technology to find sources of materials to use for their renovations, as well as, the costs for their chosen materials.

Monitoring Student Responses

- Students will keep individual records of their research that will be turned in. Students will summarize their plans after they have chosen the tiles that they think will both fit and look the best.
- Students will work together to complete calculations. They can guide each other through the processes of finding cost for each type of tile as well as figuring out the number of tiles needed to fill the spaces. Since each student will have a different selection of tiles and a different plan to cover the floor, they will have to take turns checking each other's work.
- The teacher may need to assist students in calculating the number of tiles needed to cover a given area, especially if the measurement units for the tiles differ from the units used on the blueprint.
- Students who are ready to move forward may choose to select another combination of tiles to cover the floor, using tiles with different shapes and different dimensions.

Assessment

Students will submit their cost analysis using the enclosed template to their teacher with proper citations of their sources. The teacher should decide which citation guidelines are expected (MLA, APA, etc.).

Extensions and Connections (for all students)

- Students will present their choices for materials, cost analysis, and kitchen renovations in lesson 6.
- Students may choose to present two options for the tile combinations for the floor and/or countertops. Any student choosing to do so will have a cost analysis presented for each choice.

Strategies for Differentiation

This lesson is differentiated by interest. Students are allowed to select their building materials for designing their kitchen floor and countertops. They are not limited to any specific vendor or type of building material.

Name: _____ Date: _____

Tile Shape	Brand	Store	Color	Material	Dimensions	Cost per Unit	Number Needed	Overall Cost
Rectangle	*Interceramic*	*Lowe's*	*Linen Smoke*	*Ceramic*	*16 in x 12 in*	*$1.37 / sq ft*	*10*	*$13.70*

Name: _____ Date: _____

Countertop Tiles Research

Brand	Store	Color	Material	Dimensions	Cost per Square Foot	Number of Square Feet	Overall Cost
Emser	*Lowe's*	*Mystique Quadri*	*Glass*	*4 in x 4 in*	*$28.29 / sq ft*	*10*	*$282.90*

Lesson 6: Kitchen Floor Tessellations

Strand: Geometry
Mathematical Objective(s)

The goal of this lesson is for students to design one tessellation which will be used in the kitchen remodeling project. During this lesson, students will use GeoGebra to implement their designs. The students will measure their tessellations to ensure that their designs will appropriately cover the area of the floor space. Students will present their designs with their cost analysis findings.

Mathematics Performance Expectation(s)

List all applicable Virginia College and Career Ready Mathematics Performance Expectation(s) for each lesson:

- MPE.3: The student will use pictorial representations, including computer software, constructions, and coordinate methods, to solve problems involving symmetry and transformation.

- MPE.4: The student will verify characteristics of quadrilaterals and use properties of quadrilaterals to solve real-world problems.

Related SOL

- G.3 (Symmetry and transformations)
- G.9 (Quadrilaterals)
- G.10 (Real-world problems involving angle measures, including tessellations)

NCTM Standards

- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships
    - explore relationships (including congruence and similarity) among classes of two- and three-dimensional geometric objects, make and test conjectures about them, and solve problems involving them
- Apply transformations and use symmetry to analyze mathematical situations
    - understand and represent translations, reflections, rotations, and dilations of objects in the plane by using sketches, coordinates, vectors, function notation, and matrices
- Use visualization, spatial reasoning, and geometric modeling to solve problems

    - draw and construct representations of two- and three-dimensional geometric objects using a variety of tools
    - use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture
- Understand measurable attributes of objects and the units, systems, and processes of measurement
    - make decisions about units and scales that are appropriate for problem situations involving measurement

Materials/Resources

Describe the materials and resources (including instructional technology) you plan to use in each lesson.

- Kitchen blueprint
- Computer
- GeoGebra software
- Choice of presentation materials (i.e. PowerPoint, poster board, etc.)

Introduction: Setting Up the Mathematical Task

Clearly introduce the goal of the lesson.

- In this lesson, students will tessellate polygons in order to design and renovate the kitchen floor. Students will use their prior knowledge about finding area of various polygons to ensure that their design will accurately cover the entire floor given the dimensional parameters of the kitchen. Students have already selected materials for their floor design.
- Describe planned time outline: 2 90-minute blocks or 4 45-minute periods
    - Tessellation Renovation using blueprint: 90 minutes
    - Presentations: 90 minutes

Student Exploration 1: Blueprint Tessellation

Kitchen floor Tessellation:

- Pass out blueprints and area calculations that all students turned in from lesson 3. Students have already decomposed their floor space into quadrilaterals and triangles to find the area, so this lesson's objective is to create a tessellation that will cover the space. Students will need to create a design that will cover the entire floor space without any overlapping or gaps.
- At this point, students have already chosen materials for their design. Therefore, the size and shape of the tiles is known, but the tessellation pattern has not been finalized.

- Students may begin by sketching ideas out on computer paper or graph paper. Once students are comfortable with their sketch, they should turn it in.
- Once their sketches are approved, students will finalize their designs by using GeoGebra. On GeoGebra, students will create one file for the floor design. The parameters of the space should be set up first. Then, students will apply their design to the area at whatever scale is needed for their selected tiles. From this, they will have to figure out how many of each tile they will need and if any cuts will need to be made to fit the space accurately. Students can then update their records from Lesson 5 to make sure that the count of tiles is accurate.
- By using GeoGebra, students will have to apply knowledge of transformations to keep their tiles consistently sized and appropriately tessellated.

This lesson assumes that students are familiar with creating a tessellation using various polygons, which was reviewed in Lesson 4. Also, students should be comfortable using GeoGebra in order to create a tessellation similar to their sketch (mock-up).

Students should be operating on Analysis level on van Hiele scale with regard to tessellations.

Students may have difficulty using more than one polygon to complete their tessellations. Also, students may have difficulty tessellating the floor using the blueprint since it is an odd shape, as opposed to the first exploration when the plane was rectangular (sheet of paper). Therefore, students will be allowed to work in pairs to collaborate on this activity.

Monitoring Student Responses
Students will communicate their thinking throughout this lesson by discussing it with a partner. Students that move through this activity quickly are free to design another unique tessellation using GeoGebra. Students' tessellations will be evaluated using the enclosed rubric.

Student Exploration 2: Presentation

- Students should create an outline explaining the process they have taken throughout this unit to create their renovation. This outline should be used to help create their presentation.
- Students can choose to create a PowerPoint presentation, video presentation, poster board, etc. to display their work. They will be presenting as if they are the contractor / designer talking to their client.

- The presentation should take about 5 minutes.
- Students will reveal the materials chosen for the project, the designs for both the floor and the countertops, and the cost analysis for the materials. Students will also describe any special processes that will be needed for the design to be implemented, such as needing to cut certain tiles to size.

Assessment

- Students will turn in their tessellation designs as GeoGebra files.
- Students will be assessed through their presentations to the class. There will be a rubric completed to make sure all aspects of the presentation are covered and that all materials for the project were turned in.

Name: _____ Date: _____

Kitchen Floor Tessellation Rubric

Description	Points Earned	Points Possible
Scale factor is indicated.		5
At least two different polygons are used for the tessellation: one of which is a quadrilateral.		5
The tessellation fits the parameters of the kitchen's dimensions.		5
Covered entire floor without gaps or overlaps.		5
Turned in sketch and finished GeoGebra file.		5
Total		25

Name: _____ Date: _____

Presentation Rubric

Description	Points Earned	Points Possible
Picture of floor tessellation included		5
Cost analysis charts included		5
Within budget		5
Fluidness in presenting (sufficient use of notecards, student did not read from notecards or PowerPoint slides)		5
Sources Cited		5
Presentation Creativity and Design		5
Total		30

Van Hiele Levels Overview

The van Hiele levels are a sequence of stages that describe the level of understanding an individual has in relation to geometry concepts. A student progresses through these stages by having more experiences with geometric ideas at the next level. Most students will follow the levels in sequential order with gifted students being the exception. Students that are mathematically gifted may be able to skip levels.

Level 0 - Prerecognition. A student at this level has little or no experience with this particular geometric concept.

Level 1 - Visualization. A student at this level will recognize the fundamentals but not be able to describe any particular properties. For example, a student at level 1 would be able to correctly identify a trapezoid out of a sequence of quadrilaterals, but this same student would not be able to list properties of a trapezoid.

Level 2 - Analysis. A student at this level knows properties of a particular shape but cannot distinguish which of these properties are necessary and which are sufficient for describing the object. A student at this level cannot make connections between geometric concepts. For example, a student at this level would list every property that he knows about a trapezoid: that it has four sides, that it has two bases, that it has a height determined by the distance between the two bases, that the bases are parallel, and that the two legs are not parallel, but not know which of these properties are sufficient to describe the figure as a trapezoid. Furthermore, a student at this level would not be able to recognize that all trapezoids are quadrilaterals, but not all quadrilaterals are trapezoids.

Level 3 - Abstraction. Students at this level can develop meaningful definitions and identify relationships among geometric ideas. For example, a student at this level would be able to define a trapezoid as a quadrilateral with one set of parallel sides. This student would acknowledge that all trapezoids are quadrilaterals but not all quadrilaterals are trapezoids.

Level 4 - Deduction. A student at this level can form geometric proofs and can distinguish easily between necessary and sufficient conditions. For example, a student at this level would be able to prove that quadrilateral is a trapezoid and may even be able to derive formulas in relation to a trapezoid's area.

Level 5 - Rigor. A student at this level has a sophisticated level of understanding of geometric concepts. This student fully understands deductive reasoning and is comfortable using indirect and direct proofs. This student could apply their knowledge to non-Euclidean systems. This level on the van Hiele scale typically is reserved for those

who have fully mastered geometric concepts and these individuals would be considered mathematicians.